Question 1 *(10 marks)* Use a SEPARATE writing booklet **Marks**

(a) Use $\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of $y = x^2 - 3x$. **2**

(b) Differentiate:

(i) $3x^3 - \frac{x}{2} + 7$ **1**

(ii) $\sqrt{x^2 - 1}$ **2**

(iii) $\frac{x^2}{1-3x}$. **2**

(c) Find the equation of the tangent to the curve $y = \frac{1}{x}$ at the point where $x = 2$. **3**

Question 2 *(10marks)* Use a SEPARATE writing booklet

(a) If α and β are the roots of the equation $9x^2 + 3x - 2 = 0$, find the values of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

(b) Find the values of p for which the quadratic equation $x^2 + 3x + p = 0$ has equal roots. **2**

(c) Solve for x $(2^x)^2 - 9(2^x) + 8 = 0$. **2**

(d) Solve the inequality $x^2 - 4x > 0$. **2**

Question 3 *(10marks)* Use a SEPARATE writing booklet **Marks**

The equation of a curve is given by $y = x^3 - 3x^2 + 1$.

(i) Find the coordinates of the stationary points and determine their nature. **3**

(ii) Determine if there are any points of inflexion and if so, find their coordinates. **2**

(iii) Sketch the curve for $-1 \le x \le 3$, showing stationary points and any points of inflexion. **2**

(iv) For what values of x is the curve concave down? **1**

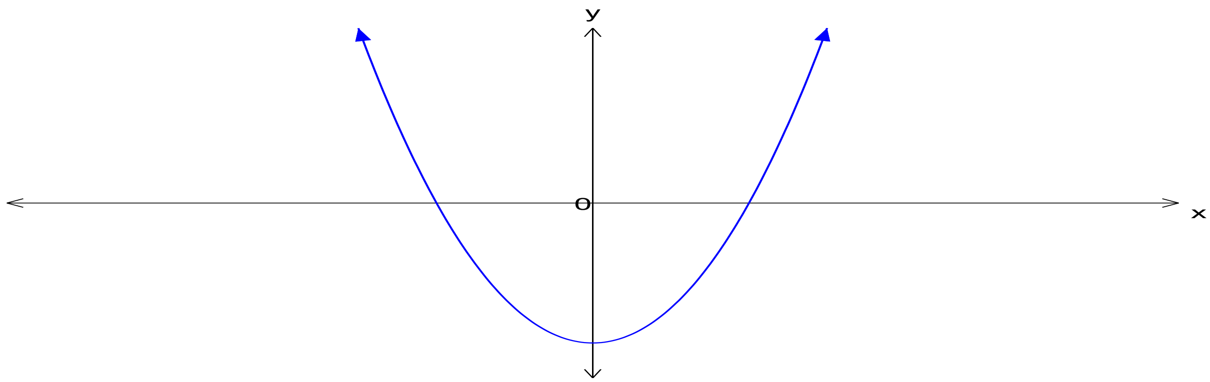
(v) Use your sketch to determine what values of k will the equation $x^3 - 3x^2 + 1 - k = 0$ have 3 distinct solutions in the domain $-1 \le x \le 3$? **2**

Question 4 *(10 marks)* **Use a SEPARATE writing booklet** **Marks**

(a) Determine the values of A, B and C in

$$x^2 - 3x + 5 \equiv A(x-1)^2 + B(x-1) + C$$ **3**

(b) Copy the sketch of $y = f(x)$. Sketch a possible graph of $y = f'(x)$ on it. **2**

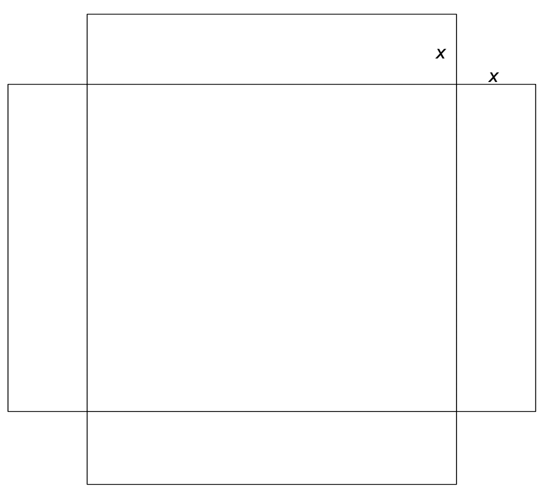


(c) A 6 m by 6 m square sheet of metal has smaller x m by x m squares cut from its corners as shown below. The sheet is bent into an open box.

(i) Use a diagram to help show that the volume of the box is given

by $V(x) = x(6-2x)^2 \ m^3$. **1**

(ii) What sized squares should be cut out to produce the box of greatest capacity? **4**



Question 1

(a) $y = x^2 - 3x$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - x^2 + 3x}{h}$$ ✓

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h - \cancel{x^2} + \cancel{3x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 3)}{\cancel{h}}$$ ✓

$$= 2x - 3$$

(b) (i) $$\frac{d\left[3x^3 - \frac{x}{2} + 7\right]}{dx} = 9x^2 - \frac{1}{2}$$ ✓

(ii) $$\frac{d\left[(x^2-1)^{\frac{1}{2}}\right]}{dx} = \frac{1}{2}(x^2-1)^{-\frac{1}{2}} \cdot 2x$$ ✓

$$= \frac{x}{\sqrt{x^2-1}}$$ ✓

(iii) $$\frac{d\left[\frac{x^2}{1-3x}\right]}{dx} = \frac{(1-3x) \cdot 2x - x^2 \cdot -3}{(1-3x)^2}$$ ✓

$$= \frac{2x - 6x^2 + 3x^2}{(1-3x)^2}$$

$$= \frac{2x - 3x^2}{(1-3x)^2}$$ ✓

(c) $y = \frac{1}{x}$

$y = x^{-1}$

$$\frac{dy}{dx} = -x^{-2}$$

$$= -\frac{1}{x^2}$$ ✓

$\therefore$ when $x = 2$, $\frac{dy}{dx} = -\frac{1}{4}$

$\therefore$ Equation of tangent

$$y - \frac{1}{2} = -\frac{1}{4}(x - 2)$$ ✓

$$4y - 2 = -x + 2$$

$$x + 4y - 4 = 0 \ \#$$ ✓

Q2

(a) $9x^2+3x-2=0$

(i) $\alpha+\beta=-\frac{1}{3}$ ✓

(ii) $\alpha\beta=-\frac{2}{9}$ ✓

(iii) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ ✓

$=\frac{1}{9}+\frac{4}{9}$

$=\frac{5}{9}$ ✓

(b) $x^2+3x+p=0$

$\Delta=0$ equal roots

$\therefore b^2-4ac=0$ ✓

$9-4p=0$

$p=\frac{9}{4}$ ✓

(c) $(2^x)^2-9(2^x)+8=0$

$(2^x-8)(2^x-1)=0$

$2^x=8$ or $2^x=1$

$x=0,3$ ✓✓

(d) $x^2-4x>0$

$x(x-4)>0$

$\therefore x>4$ or $x<0$

✓ ✓

Q3 $y = x^3 - 3x^2 + 1$

(i) $\frac{dy}{dx} = 3x^2 - 6x$

let $\frac{dy}{dx} = 0$ to find st pts

$3x(x-2) = 0$

$x = 0, 2$

$\therefore$ St pts at $(0,1)$ and $(2,-3)$ ✓✓ Both coordinates.

$\frac{d^2y}{dx^2} = 6x - 6$

when $x = 0$ $\frac{d^2y}{dx^2} < 0$ $\therefore$ Max at $(0,1)$ ✓(Test)

when $x = 2$ $\frac{d^2y}{dx^2} > 0$ $\therefore$ Min at $(2,-3)$

(ii) let $\frac{d^2y}{dx^2} = 0$ to find pt of inflexion

$6x - 6 = 0$

$x = 1$ $\therefore$ at $(1,-1)$ ✓

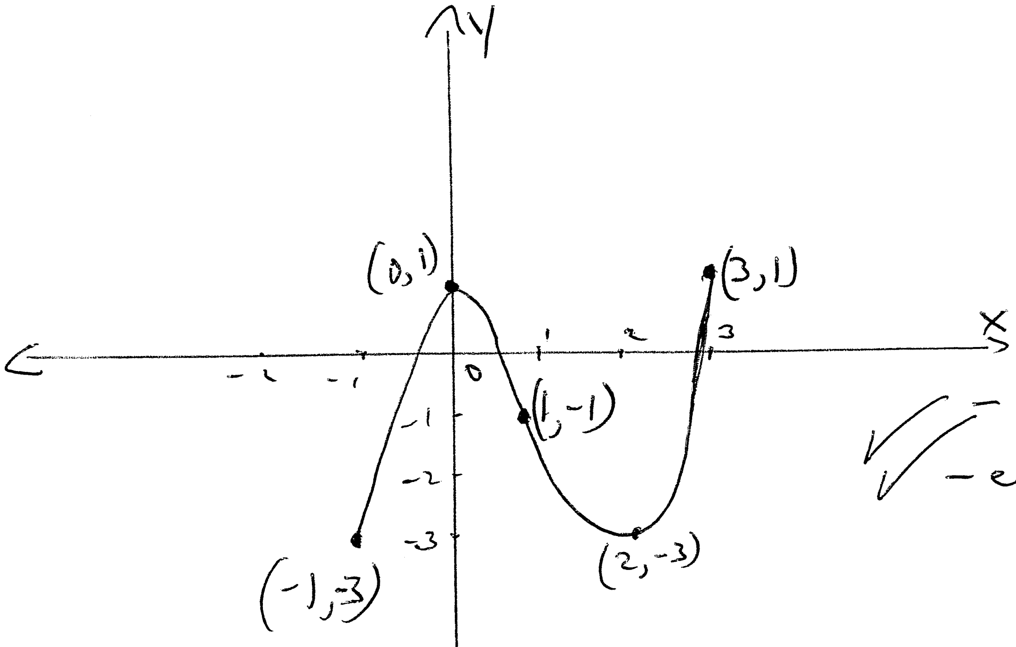
Concavity test ✓

x	0.9	1	1.1
$\frac{d^2y}{dx^2}$	$-$	0	$+$

$\therefore$ change in concavity

(iv) $x < 1$ ✓ Concave down $\left[\frac{d^2y}{dx^2} < 0\right]$

(v)



✓ – all others

✓ – endpts.

(vi) let $x^3 - 3x^2 + 1 = k$ ✓

$\therefore -3 < k < 1$ ✓

Q4 (a) $x^2-3x+5 \equiv A(x-1)^2+B(x-1)+C$

$A=1$ [coeff of x^2 is 1]

let $x=1$ ∴ $1-3+5=C$

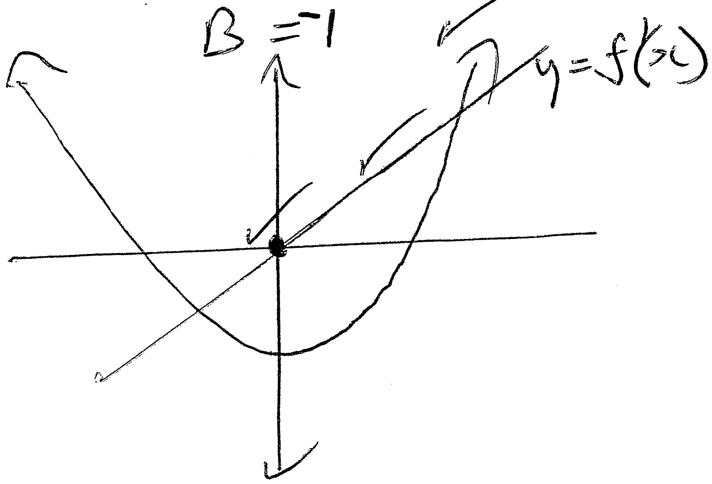
$C=3$

let $x=0$ ∴ $5=A-B+C$

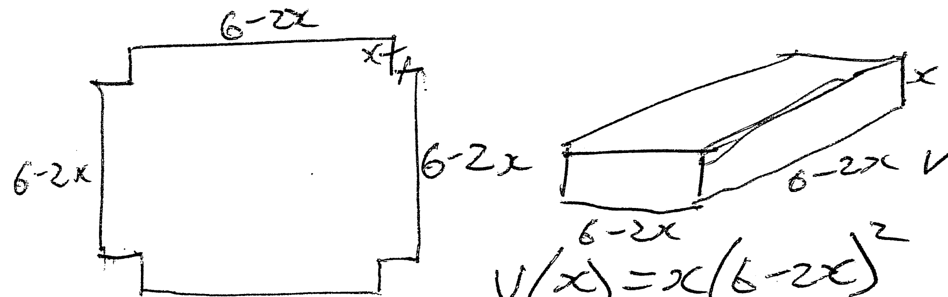
∴ $5=1-B+3$

$B=-1$

(b)



(c) (i)



$V(x)=x(6-2x)^2$

(ii) $V(x)=x(6-2x)^2$

$V'(x)=(6-2x)^2-4x(6-2x)=(6-2x)^2-24x+8x^2$

$=(6-2x)[6-2x-4x]$

$=(6-2x)[6-6x]$

∴ $x=3,1$

∴ Max when $x=1$

∴ squares 1m by 1m #

$V''(x)=-4(6-2x)-24+16x$

$V''(1)=-4\times4-24+16<0$ $V''(3)=-4(6-6)-24+48>0$